# Square Trains Task H.O.T. Lesson LP 

Discourse Team - For week of September 24

## Unit: Patterns \& Functions

## Materials needed:

Square tiles (approx 15 per group)
Role cards (including side wall with teams and roles for easy reference)
Task card - 1 per student
Scrap paper - put at least 1 piece of scrap paper in the middle of the table so that students can work together in the middle as needed.
Exit slip (if desired)
Calculator (optional)
Core mathematics - what are we working on when we have students do this task?

- Connecting the algebraic expression to the geometry - being able to explain how they "saw" the perimeter and connect it to the algebraic expression
- Modeling a situation with an algebraic equation
- Problem solving/determining a way to organize data (strategic competence)
- Building on order of operations (the need this to write expressions properly; to show they are equivalent)
- To build understanding of the meaning of symbols and a sense of perimeter as a function ${ }^{1}$

Language: we did not discuss much
Vocab of perimeter, train, (square).
Students will also be working on explaining their thinking and recording in words the process they use to figure out the perimeter of a square train of any length.

## Square Trains Task:

## Square Trains



Figure 1


Figure 2


Figure 3

## Begin lesson

[^0]Review group norms, group roles. Make sure students understand what they are responsible for individually and as a group.
Emphasize that the goal is not just to get an answer, but to be able to explain your thinking.
Reinforce value of working in groups.
Perhaps review group questions, or explain the idea of check points.

## Launch task:

Use squares - show first 1 square and then a train of 2 squares.

1) for these squares, each side has length 1.
2) show these to give them the idea of trains
3) review/make sure they know what the perimeter is
4) discuss how to count/determine the perimeter for the train of 2 squares (for 2 squares, it's not 8 . Why?)

Rationale: to get into the problem, they need to be clear on what perimeter is, what counts as part of the perimeter when the figures at put together (make a train), and the idea of that they are making a train

Hand out task cards. Tell groups approximately how long they have to work on the task. Monitor and make sure the task is being read outloud. Explain how the materials manager is to get the square tiles (or already have those on the desks).

Monitor groups as students work. Offer positive feedback for group behaviors that are productive.

Respond when the materials manager has hand raised for a group question (make sure it's a group question before you respond) or for a Check Point.

## Check point 1: (explain how they found perim of train with $\mathbf{8 0}$ squares)

Questions: ask students to explain how they arrived at their answer, whether correct or incorrect.
Listen for them to talk about the number of sides per square that contribute to the perimeter. Listen for them to distinguish the two end squares from the rest (or just add in the end edges).
Listen for students to use mathematical language - prompt them to be precise and say perimeter and number of square tiles.

Possible follow up questions:
Ask another student in the group to repeat the explanation
Ask another student to explain how their group would then do it for a square train with 100 square tiles. (This will help move them towards a generalization.)

Some students may use an argument that is based on knowing the result for smaller trains. For example, a group might explain that for a square train of 4 square tiles it had a perimeter of 10 , and so to get to 80 squares, you need to add 76 squares, and it's 2 more
per square, so you add 152 for a total of 162 . This is good logic, but may be harder to generalize (as it is a recursive formula). You might ask if there's a way they can do it without relying on the 4 -square train. Perhaps they don't know, or forgot. Or you could ask them for a 100 -square train, and then a 350 -square train, which might motivate them to think about how they could find it direction. ((Also, $2(n-4)+10$ is perfectly valid if they can explain it.))

When the student(s) in a group can explain to your satisfaction, give them a stamp or points. (We did not decide on how many points.)

If students cannot explain, say that you'll come back in a few minutes. Let the group talk it over for a bit.

Check point 2: Explain in words how you can find the perimeter for any sized square train. Try to write an expression for the perimeter using mathematical symbols.

## Different ways students would do it

- Idea of $2 n+2$. These students might see that they have the "top" and the "bottom" of all of the squares (all n), but that the 2 end squares each have 1 more edge that has to be counted (this creates the +2 )
- Idea of $2(n-2)+6$. These students might see that all except the end 2 squares ( $n$ -2 ) have 2 sides that "count" (the top and bottom). The 2 end squares have 3 sides showing each, which contribute 3 units to the perimeter.
- Building on Diana's way before, students might try to start with 4 n because each square has 4 sides. They'll want to have $4 n-$ $\qquad$ . If they make a table, they might see that the pattern is $4(1), 4(2)-2,4(3)-4,4(4)-6$, etc. They could see that, for each figure, there are $(\mathrm{n}-1)$ place where 2 sides come together, so the formula is $4 n-(n-1) * 2$.
- Idea of the " +2 ". Many students will likely see this pattern, and perhaps want to write $\mathrm{P}=\mathrm{n}+2$ or something like that. Building on this, the recursive formula: Perimeter of figure $n=$ Perimeter of figure $(n-1)+2$. (or $\mathrm{P}_{\mathrm{n}}=\mathrm{P}_{(\mathrm{n}-1)}+2$, where P $\mathrm{P}_{0}=4$ )
- Other ways?


## Follow up questions:

What you ask may depend on whether they've gotten a symbolic form or not. Regardless, you can ask about their thinking and make sure they can connect their process of finding the perimeter with the physical features of the square train.

Possible questions:
Where does your " +2 " come from? Can you show me that with the square trains? (Alt: where's your +6 from.)
Why do you multiply the number of square tiles by 2 ? How does that make sense?
How come, when I add a square with 4 sides each time, that I only add $\underline{2}$ to my perimeter? Why don't I add 4 ? (Or at least 3, as 3 new sides seem to be added on for each one I add on the end.)

If students cannot explain, say that you'll come back in a few minutes. Let the group talk it over for a bit.

## End groupwork time:

Have materials managers collect supplies.
Whole-class Debrief of Task: We have not discussed this much - teacher's choice. There can be a full group debrief. One option is to put up the formulas of different groups and talk about each one in relation to a picture of the square trains, and/or talk about how they compare and contrast. How were different groups seeing the pattern? (This can get into order of operations. Is $2(n-2)+6$ the same as $2 n+2$ ? How do you know? Could I leave out the parentheses for $2(\mathrm{n}-2)+6$ ? Why/why not?)

## Closure/Exit slip: We have not discussed this much - teacher's choice.

Highlight the key points of the lesson - students work on explaining their ideas; they worked on working together.
Could have an exit slip: could ask students to reflect on how they did with their role, or how their group worked together, or record what they understand and what they still have questions on.

Ask students to put all their papers in a folder on the table. Ask materials manager to hand those to you (or leave on desk and you'll collect....)

Other notes on the lesson:
To promote interdependence of groups:

- Review group norms at beginning
- Reinforce throughout lesson
- Group check in; use stamps or give points; have 1-2 check in points.
- Anything else? Emphasize a particular norm? group questions?

Another Extension (there are 2 written on the bottom of the task card)
Have students use the same reasoning, or similar reasoning, and determine the perimeter for a square train, a pentagon train. See if they can see how the 3 perimeter formulas are related.

There is an interesting discussion of this problem (well, a very similar problem) at http://intec.concord.org/tools/al/findingpattern.html This might help give us some ideas and anticipate the implementation.

## Next day follow up:

Vanessa suggested giving students 3 different explanations the next day (of how they found the perimeter for an 80 -square train, or how the formula made sense) and having students read the explanations and talk about whether they were good explanations or not, what made them good, etc. This could be the writing-to-learn piece.


[^0]:    ${ }^{1}$ I was poking around on the web for a good hexagon picture and found this "goal" for this task on this website: http://intec.concord.org/tools/al/a1read_hand.html

